

AP CALCULUS AB - SUMMER WORK

Name: _____

The first test of the year will take place immediately upon your return to school in the fall and will cover pre-calculus topics. To prepare yourself for the test, you should complete all work in this packet and be sure you understand the topics listed below. Your work will be collected during that test period. The test will not be calculator active, and therefore you should practice completing all of the problems in this packet without a calculator.

Pre-calculus skills you need to know:

Algebra skills:

Solve basic 1-variable equations and inequalities, including proportions
Work with algebraic fractions – finding common denominators when necessary
Linear equation work: slope & its various formulas, meanings, & applications, y-int. & its meaning, x-int./root/zero
Finding and then writing (in an appropriate form) the equation of a line given 2 points or given a point and a slope
Parallel & perpendicular lines and their slope properties
Know these three forms of a linear equation: slope-intercept, point-slope, standard
Quadratic equations and their graphs
Solve a quadratic equation by: factoring, quadratic formula, completing the square, & graphing calculator
Parabolas & circles
Solve a system of 2 equations to find their common (x, y) solution(s)
Radicals – simplification rules, basic operations (add, subtract, multiply & divide), and graphs
Square roots and cube roots
Solving polynomial equations (third and fourth degree): factoring & synthetic substitution methods
Functions & their uses: notation, inverse functions, operations (add, sub., mult., & div.) & compositions
Transformations and their effects on equations– translations, reflections & rotations
Solve basic absolute value equations/inequalities and then graph the results
Sequences & series
Exponential & logarithmic functions
Basic graphs and properties

Geometry skills

Basic and specific vocabulary – tangents, parallels, chords, secants, special quadrilaterals
Pythagorean Theorem, both on and off the x-y grid
Basics of a mathematical proof & the logic necessary to follow the proof of a theorem
Similar triangle properties - scaling factors, setting up a proportion, direct & inverse variation
Basic perimeter, circumference, area, surface area, & volume formulas
Plotting on an x-y grid: 2-d shapes & equations (functions)
Basic linear and angular properties of special 2-d shapes: triangles & quadrilaterals

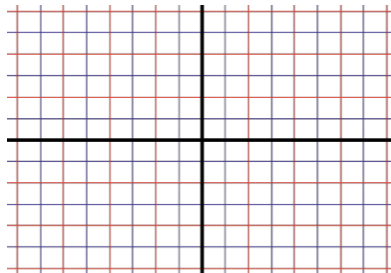
Trigonometry skills

Solve right triangle trig problems using the sine, cosine, and tangent ratios
Six trig ratio definitions & basic trig identities: reciprocal, quotient, Pythagorean and double-angle identities
Radian measure and the unit circle
Radian measure for these special degree angles: 0, 30, 45, 60, 90, ..., 315, 330, 360
Trig values for the special angles (in radians)
Build and use the special right triangles on an x-y grid & on the unit circle
Graphs of sine, cosine, & tangent functions (in radians)
Solving basic one-step and two-step trig equations (in radians) over a restricted domain

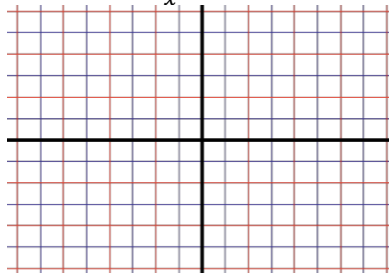
Sketch each of the following; label all asymptotes and intercepts.

Although you may use a graphing calculator to help you graph these functions initially, by the start of school in the fall, you should have these graphs memorized as we will be working with transformations of these functions often and the AP expects that you know them without your calculator.

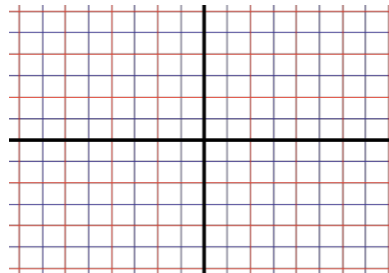
1. $y = x^3$



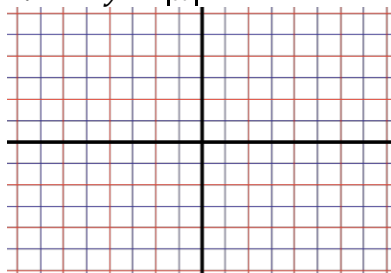
2. $y = \frac{1}{x}$



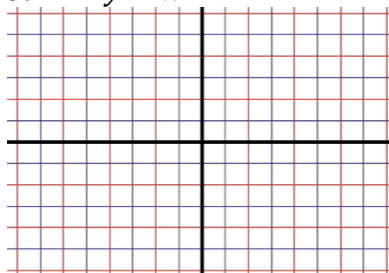
3. $y = \sqrt{x}$



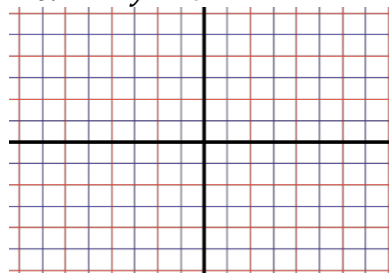
4. $y = |x|$



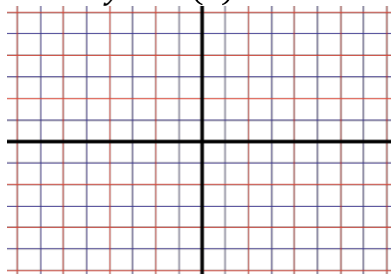
5. $y = x^2$



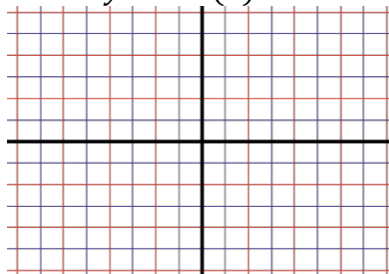
6. $y = e^x$



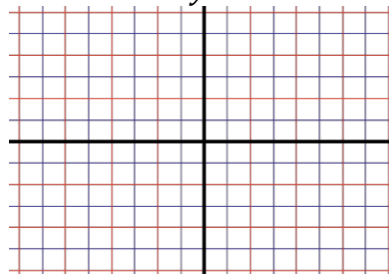
7. $y = \ln(x)$



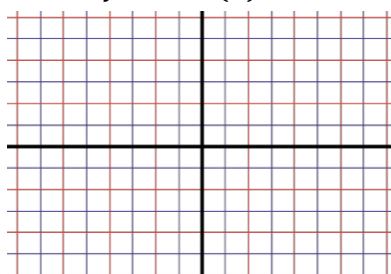
8. $y = \sin(x)$



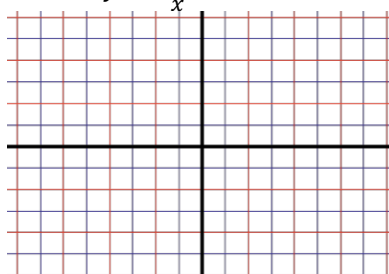
9. $x = y^2$



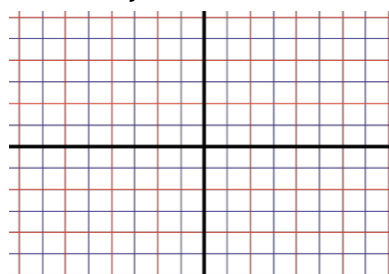
10. $y = \cos(x)$



11. $y = \frac{|x|}{x}$



12. $y = \sqrt{9 - x^2}$



Evaluate without a calculator:

13. $8^{\frac{2}{3}} = \underline{\hspace{2cm}}$

14. $81^{\frac{3}{4}} = \underline{\hspace{2cm}}$

15. $x^{-1} = 0.5$, $x = \underline{\hspace{2cm}}$

16. $x^{\frac{2}{3}} = 9$, $x = \underline{\hspace{2cm}}$

17. $x^{\frac{3}{4}} = 8$, $x = \underline{\hspace{2cm}}$

18. $100^x = 0.1$, $x = \underline{\hspace{2cm}}$

19. $2^x = 0.25$, $x = \underline{\hspace{2cm}}$

20. $e^{\ln \pi} = \underline{\hspace{2cm}}$

21. $\ln(e^x) = \underline{\hspace{2cm}}$

22. $\ln(e\sqrt{e}) = \underline{\hspace{2cm}}$

23. $\log_4(32) = \underline{\hspace{2cm}}$

24. $\log_3(\sqrt{3}) = \underline{\hspace{2cm}}$

25. $\frac{1}{2} \log(100) = 3 \log(2) + \log(x) - \log(4)$, $x = \underline{\hspace{2cm}}$

26. $(y^{3x})(y^2) = \frac{y^{14}}{y^{-6}}$, $x = \underline{\hspace{2cm}}$ $[y \neq 0]$

27. $\frac{\sqrt{x}}{7} = x^y$, $y = \underline{\hspace{2cm}}$

28. $\frac{x^{-\frac{7}{2}}}{x^{\frac{1}{2}}} = x^y$, $y = \underline{\hspace{2cm}}$

29. $\frac{1}{\sqrt[5]{x}} = x^y$, $y = \underline{\hspace{2cm}}$

30. A circle has the equation $x^2 + y^2 + 6x - 8y - 11 = 0$. Find the radius and the coordinates of the center.

31. Find an equation for the perpendicular bisector of the line segment whose endpoints are $(-4, 3)$ and $(8, -5)$.

32. Find all real values of x that satisfy the equation, $|x + 2| = 2 - |x|$.

In the following decomposition problems, $f(x) \neq g(x)$, $g(x) \neq x$ and $h(x) \neq x$.

33. If $f(g(x)) = (2x - 3)^2$, then $f(x) = \underline{\hspace{2cm}}$ $g(x) = \underline{\hspace{2cm}}$

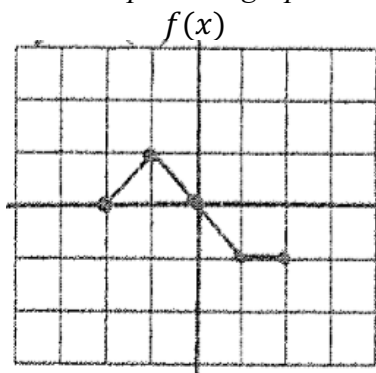
34. If $f(g(x)) = \sin^2 x$, then $f(x) = \underline{\hspace{2cm}}$ $g(x) = \underline{\hspace{2cm}}$

35. If $f(g(x)) = \ln(x^2 + 1)$, then $f(x) = \underline{\hspace{2cm}}$ $g(x) = \underline{\hspace{2cm}}$

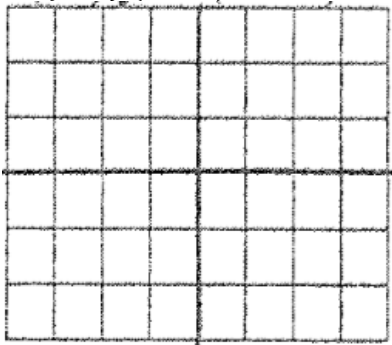
36. If $f(g(x)) = 2^{\cos x}$, then $f(x) = \underline{\hspace{2cm}}$ $g(x) = \underline{\hspace{2cm}}$

37. If $f(g(h(x))) = |\tan(\log(x))|$, then $f(x) = \underline{\hspace{2cm}}$ $g(x) = \underline{\hspace{2cm}}$ $h(x) = \underline{\hspace{2cm}}$

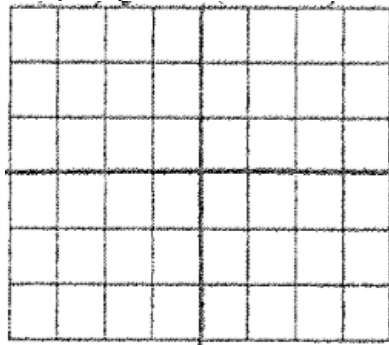
Use the graph of $y = f(x)$, shown below, to produce graphs that correspond to the given functions:



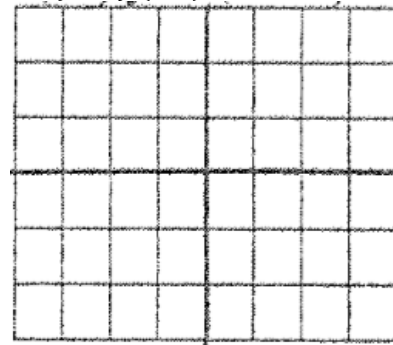
38. $y = f(x + 1)$



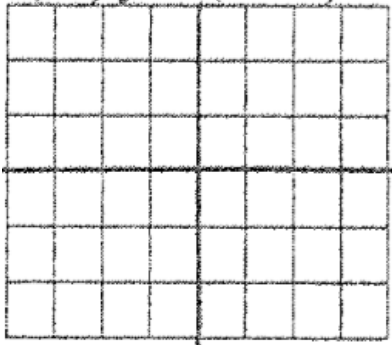
39. $y = f(x) + 1$



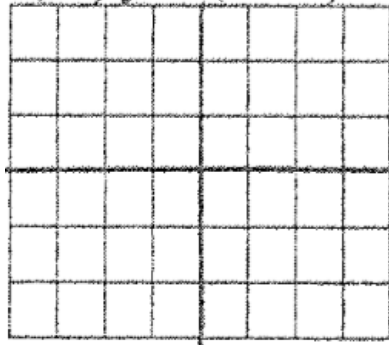
40. $y = f(-x)$



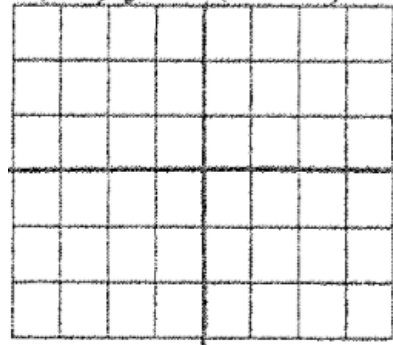
41. $y = -f(x)$



42. $y = f(-x) - 1$



43. $y = |f(x)|$



Find the exact values for the following:

44. $\sin\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \underline{\hspace{2cm}}$

45. $\cos\left(\sin^{-1}\left(\frac{8}{17}\right)\right) = \underline{\hspace{2cm}}$

46. If $\cos(x) = 0.4$, then $\tan(x) = \underline{\hspace{2cm}}$.

47. Find the solution set for each of the following inequalities: $\frac{x^2+2x-3}{x^2-4} > 0$

49. A utility company has a fleet of vans. The annual operating cost C (in dollars) of each van is estimated to be $C = 0.50m + 2500$ where m is measured in miles. The company wants the annual operating cost of each van to be less than \$10,000. To do this, m must be less than what value?

50. Write the standard form of the equation of the circle and sketch its graph:

a) $x^2 + y^2 - 2x + 6y + 6 = 0$

b) $x^2 + y^2 - 4x + 2y + 3 = 0$

51. Without a calculator, sketch a graph of the rational function whose equation is:

$$y = \frac{x^2 - 5x - 6}{x^2 + 5x - 6}$$

Clearly label or identify all axis intercepts, and all vertical and horizontal asymptotes.



52. Solve the following equation for y in terms of x: $x^2 - 2x = 2y + y^2$

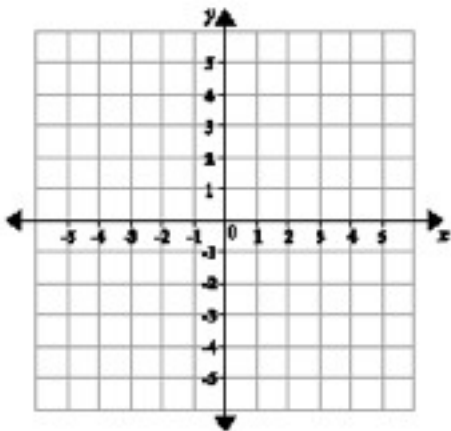
53. Simplify : $\frac{1+\cos x}{\sin^2 x + \cos x + \cos^2 x}$

54. Find an equation for the line tangent to the circle, whose equation is $x^2 - 4x + y^2 - 6y = 5$ if the point of tangency is (5,6).

55. Write the equation of the line going through the points (-3, 5) and (2, -10) in each of the following forms: slope-intercept, point-slope and standard.

56. A right circular cylinder, with radius x inches, has a volume of 150 cubic inches. Define the surface area of the cylinder as a function of x.

57. Sketch and shade the region bounded by the graphs of $y = x^2 - 3$ and the line $y = x$.



58. Find all intercepts of the graph of $y = \frac{x+2}{x-3}$.

59. Find all points of intersection of the graphs of $x^2 - 2x - y = 6$ and $x - y = -4$.

60. Find an equation for the line passing through the point (4, -1) and perpendicular to the line $2x - 3y = 3$.

61. Find the domain of $f(x) = \frac{1}{\sqrt{3-2x}}$.

62. Find $f(x + \Delta x)$ for $f(x) = x^3 + 1$

63. If $f(x) = \frac{1}{\sqrt{x}}$ and $g(x) = 1 - x^2$, find $f(g(x))$.

64. If the point $(-3, \frac{1}{2})$ lies on the graph of the equation $2x + ky = -11$, find k .

65. Given $f(x) = x^2 - 3x + 4$, find $f(x + 2) - f(2)$.

66. Let $f(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$. Find $f(3)$. _____ Find $f(0)$. _____ Find $f(-2)$. _____

67. In order for a company to realize a profit in the manufacture and sale of a certain item, the revenue, R , for selling x items must be greater than the cost, C , of producing x items. If $R = 80x$ and $C = 60x + 1600$, for what values of x will this product return a profit?

68. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2-4}}{x-3}$?

69. If the graph of $y = \frac{ax+b}{x+c}$ has horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then what does $a + c$ equal?

70. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, find $h(x)$.

71. If $f(x) = \frac{x}{x+1}$, find the inverse function, $f^{-1}(x)$.

72. Evaluate the following: (you should know these without a calculator!)

a) $\cos\left(\frac{\pi}{2}\right) =$

b) $\sin\left(\frac{\pi}{4}\right) =$

c) $\tan\left(\frac{\pi}{3}\right) =$

d) $\sin\left(\frac{\pi}{6}\right) =$